Talent, Labor Quality, and Economic Development
Appendix For Online Publication

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1 Derivation of First-Order Conditions

We present below the derivation of first-order conditions for the model presented in the text. The reader should recall that individual earnings at t are given by $W_{U,t}z$ when unskilled, and $W_{S,t}h_t z$ when skilled. Augmented skills, h, are produced via $h_{t+1} = Bx_t^{\phi}$.

The household problem is to choose sequences $\{C_t, K_{t+1}, x_t, \hat{z}_t\}_0^{\infty}$ to maximize

$$\max \sum_{t=0}^{\infty} \beta^t L_t \ \log(\frac{C_t}{L_t}), \tag{1}$$

subject to:

$$\begin{aligned}
C_t + I_t + N_t(1 - G(\hat{z}_t))x_t &= W_{U,t}U_t + W_{S,t}S_t + R_t K_t, & (2)\\
K_{t+1} &= K_t(1-\delta) + \frac{I_t}{p_t}, & (3)\\
U_t &= U_{t-1} + N_t \int_0^{\hat{z}_t} z g(z) dz, & (4)\\
S_t &= S_{t-1} + N_{t-1} h_t \int_{\hat{z}_{t-1}}^{\bar{z}} z g(z) dz, & (5)\\
h_{t+1} &= B x_t^{\phi}. & (6)
\end{aligned}$$

Sustituting the constraints (3-6) in the resource constraint (2), the Lagrangian function for the problem is:

$$\begin{aligned}
\mathcal{L} \ &= \ \sum_{t=0}^{\infty} \beta^t \{ L_t \log(\frac{C_t}{L_t}) + \lambda_t \ [W_{U,t}(U_{t-1} + N_t \int_0^{\hat{z}_t} z g(z) dz)\\
&+ \ W_{S,t} \ (S_{t-1} + N_{t-1} B x_{t-1}^{\phi} \int_{\hat{z}_{t-1}}^{\bar{z}} z g(z) dz)\\
&+ \ R_t K_t - p_t(K_{t+1} - K_t(1-\delta))\\
&- \ C_t - N_t(1 - G(\hat{z}_t))x_t] \}
\end{aligned}$$

The corresponding first-order conditions are:

$$C_t: \quad \frac{L_t}{C_t} - \lambda_t = 0, \tag{7}$$

$$K_{t+1}: \quad -\lambda_t p_t + \lambda_{t+1}\beta[R_{t+1} + p_{t+1}(1-\delta)] = 0, \tag{8}$$

$$x_t: \quad -\lambda_t \ (1 - G(\hat{z}_t)) + \lambda_{t+1}\beta \ W_{S,t+1} B\phi x_t^{\phi-1} \int_{\hat{z}_t}^{\bar{z}} zg(z)dz = 0, \tag{9}$$

$$\hat{z}_t: \quad \lambda_t W_{U,t}\hat{z}_t g(\hat{z}_t) + \lambda_t g(\hat{z}_t)x_t - \lambda_{t+1}\beta \ W_{S,t+1} Bx_t^{\phi}\hat{z}_t g(\hat{z}_t) = 0. \tag{10}$$

Substituting out λ_t and rearranging, we get the three first-order conditions in the text:

$$\frac{p_t}{C_t/L_t} = \frac{\beta \ R_{t+1} + p_{t+1}(1-\delta)}{C_{t+1}/L_{t+1}}, \tag{11}$$

$$\frac{W_{U,t}\hat{z}_t + x_t}{C_t/L_t} = \beta \ \frac{W_{S,t+1}Bx_t^{\phi}\hat{z}_t}{C_{t+1}/L_{t+1}} \tag{12}$$

$$\frac{1 - G(\hat{z}_t)}{C_t/L_t} = \beta \ \frac{W_{S,t+1}\left(\int_{\hat{z}_t}^{\bar{z}} zg(z)dz\right) B\phi x_t^{\phi-1}}{C_{t+1}/L_{t+1}}. \tag{13}$$

These equations, in conjunction with the resource constraint fully determine the optimal path of the variables of interest. Equation (11) is the standard Euler equation for capital – equation (7) in the text. Equation (12) defines the division of a cohort between unskilled and unskilled workers. This is equation (8) in the text. Finally, equation (13) is the intertemporal optimality condition for investing goods in augmenting skills. This is equation (9) in the text.

2 Additional Information on Tables and Parameters

We present below two tables, 1 and 2, that convey information on data and parameters for each country used in our quantitative exercises. In Table 1, column g_L pertains to the country-specific population growth rates.[1] Columns θ and κ show the Gamma-distribution parameters that reproduce the PISA distribution in each country. Column γ reports enrollment rates from PISA data. Column p shows the price of investment goods relative to consumption goods from Penn World Tables. The last two columns show the GDP per worker and fraction of unskilled workers.

In Table 2, the first column pertains to TFP levels (A) in our experiment *Variation in TFP and PISA Scores*. The second column shows the levels of TFP (A) and the relative efficiency of good investments in augmenting talent (B) in our experiment *Matching the Division of Labor*.

[1] We set $g_L = 0$ when the reported population growth rate is negative.

Table 1: Data and Parameters For Cross-country Analysis

	g_L	θ	κ	γ	p	GDP Per Worker	Fraction Unskilled
Kyrgyz Republic	0.013	19.71	16.80	0.80	1.92	4543.74	0.87
Indonesia	0.014	13.31	27.89	0.74	1.07	7405.97	0.98
Albania	0.000	21.83	17.29	0.77	1.01	13289.95	0.93
Peru	0.012	22.16	16.47	0.84	0.86	13331.50	0.80
Thailand	0.007	14.94	28.01	0.80	0.96	13510.99	0.88
Colombia	0.012	14.93	25.51	0.65	0.91	15002.60	0.92
Jordan	0.032	17.69	21.86	0.91	0.81	15490.64	0.81
Brazil	0.014	17.09	22.57	0.81	0.86	15579.86	0.93
Tunisia	0.010	16.18	22.96	1.00	0.84	16337.12	0.88
Serbia	0.000	18.59	23.80	0.88	1.02	19764.66	0.89
Kazakhstan	0.001	17.18	23.56	0.93	1.19	20478.75	0.78
Uruguay	0.003	19.58	21.79	0.80	0.86	20519.00	0.92
Panama	0.017	18.07	19.91	0.75	0.98	20635.01	0.79
Romania	0.000	14.67	29.10	1.00	1.08	21250.18	0.88
Bulgaria	0.000	22.89	18.70	0.88	1.20	23944.00	0.84
Argentina	0.010	22.52	17.23	0.92	1.05	24554.44	0.88
Russia	0.000	15.42	30.33	1.00	1.25	27164.02	0.44
Chile	0.011	15.25	27.62	0.92	0.81	28931.14	0.73
Latvia	0.000	13.00	37.08	0.98	1.30	29709.77	0.79
Mexico	0.012	14.89	28.11	0.66	1.11	30493.25	0.83
Turkey	0.015	19.58	22.76	0.64	0.97	30978.41	0.91
Lithuania	0.000	16.27	29.29	0.85	1.37	33453.48	0.74
Poland	0.000	15.78	31.36	0.98	1.05	34392.02	0.85
Croatia	0.002	16.91	27.20	0.95	0.95	35292.10	0.91
Estonia	0.000	12.84	39.89	0.99	1.10	37788.60	0.73
Slovakia	0.000	18.59	26.72	0.99	1.13	38064.57	0.87
Portugal	0.004	17.16	28.37	0.93	0.81	39074.87	0.89
Hungary	0.000	17.30	28.34	0.98	1.12	41493.83	0.84
Czech Republic	0.000	17.62	27.97	0.95	1.05	44482.12	0.89
South Korea	0.005	14.57	37.48	0.98	0.79	50158.04	0.60
Trinidad	0.000	23.85	17.36	0.92	0.78	50742.56	0.95
Slovenia	0.000	18.11	27.69	0.96	0.87	52656.43	0.83
New Zealand	0.010	17.82	29.13	0.95	1.05	53298.97	0.49
Spain	0.014	16.99	28.46	0.98	1.03	58921.53	0.76
Japan	0.001	16.74	31.61	0.98	0.88	62583.77	0.63
Greece	0.002	17.17	27.15	1.03	0.87	63661.34	0.76
Israel	0.020	24.25	18.43	0.91	0.80	63820.47	0.69
Switzerland	0.005	18.44	28.96	0.99	0.89	65903.68	0.83
Germany	0.000	18.86	27.19	1.00	0.92	66388.95	0.82
Finland	0.002	12.58	42.96	1.00	0.89	68488.49	0.76
Denmark	0.003	15.04	33.46	0.98	0.85	68551.48	0.79
Canada	0.008	14.54	36.23	0.99	0.94	69043.75	0.65
Sweden	0.002	17.81	27.76	1.00	0.94	70656.33	0.76
Netherlands	0.006	15.08	34.86	1.00	1.03	70705.34	0.77
Hong Kong-China	0.007	16.38	33.86	0.92	0.69	70985.23	0.84
France	0.006	20.48	24.26	0.98	0.99	71336.49	0.80
Italy	0.004	17.93	26.94	0.98	0.84	71916.25	0.90
United Kingdom	0.005	15.44	31.90	1.00	0.95	72447.71	0.76
Iceland	0.010	16.35	31.00	1.00	0.80	73979.41	0.70
Austria	0.002	18.61	26.65	0.94	0.96	75466.17	0.88
Australia	0.012	17.17	29.96	0.94	0.93	77002.50	0.66
Ireland	0.017	15.05	32.36	0.98	1.05	77185.06	0.69
Belgium	0.002	21.05	24.48	1.00	0.82	79858.48	0.76
United States	0.010	17.58	27.70	1.00	0.86	85922.95	0.48
United Arab Emirates-Dubai	0.048	21.60	20.96	0.98	0.88	86315.84	0.86
Macao-China	0.026	13.85	37.92	0.80	0.98	89337.24	0.84
Singapore	0.017	19.40	28.97	0.99	0.72	90475.84	0.82
Norway	0.005	14.63	34.04	0.99	0.90	95734.87	0.74
Luxembourg	0.013	19.46	25.14	0.96	0.87	119373.72	0.85

Table 2: Parameters A and B in Experiments

	A	A	B
Kyrgyz Republic	0.30	0.39	0.37
Indonesia	0.32	0.59	0.11
Albania	0.42	0.64	0.23
Peru	0.40	0.48	0.48
Thailand	0.41	0.56	0.32
Colombia	0.44	0.65	0.24
Jordan	0.43	0.52	0.45
Brazil	0.44	0.67	0.22
Tunisia	0.44	0.61	0.31
Serbia	0.51	0.72	0.28
Kazakhstan	0.55	0.66	0.49
Uruguay	0.49	0.74	0.23
Panama	0.53	0.64	0.48
Romania	0.54	0.75	0.30
Bulgaria	0.59	0.77	0.38
Argentina	0.59	0.82	0.30
Russia	0.63	0.61	1.26
Chile	0.58	0.67	0.56
Latvia	0.67	0.82	0.46
Mexico	0.66	0.86	0.38
Turkey	0.63	0.95	0.24
Lithuania	0.72	0.85	0.53
Poland	0.67	0.90	0.34
Croatia	0.66	1.03	0.23
Estonia	0.71	0.83	0.54
Slovakia	0.72	1.03	0.29
Portugal	0.66	0.97	0.26
Hungary	0.75	1.01	0.35
Czech Republic	0.76	1.15	0.25
South Korea	0.72	0.77	0.78
Trinidad	0.76	1.39	0.14
Slovenia	0.78	1.04	0.36
New Zealand	0.83	0.82	1.05
Spain	0.88	1.07	0.47
Japan	0.85	0.72	0.71
Greece	0.87	1.07	0.46
Israel	0.85	0.97	0.60
Switzerland	0.87	1.16	0.36
Germany	0.88	1.18	0.36
Finland	0.88	1.09	0.47
Denmark	0.88	1.13	0.41
Canada	0.91	1.01	0.65
Sweden	0.93	1.15	0.45
Netherlands	0.94	1.18	0.44
Hong Kong-China	0.82	1.14	0.33
France	0.94	1.22	0.40
Italy	0.91	1.42	0.23
United Kingdom	0.95	1.17	0.46
Iceland	0.90	1.09	0.56
Austria	0.96	1.43	0.27
Australia	0.96	1.07	0.64
Ireland	1.01	1.17	0.56
Belgium	0.93	1.15	0.46
United States	1.00	1.00	1.00
UAE-Dubai	1.03	1.48	0.29
Macao-China	1.06	1.48	0.31
Singapore	0.94	1.27	0.35
Norway	1.07	1.31	0.47
Luxembourg	1.18	1.69	0.30

3 Talent-specific Investments

In our model, skill augmenting investments are common to all household members who become part of the skilled pool. One potential concern is whether by ignoring skill-augmenting investments that depend on individual talent, we are understating the impact of TFP on output per worker and thus, obtaining a TFP-elasticity in our sample that is lower than otherwise. Our analysis below shows that this concern is unwarranted. Specifically, we show that a properly calibrated version of the model with talent-specific investments implies similar TFP differences across countries and a similar TFP-elasticity of output per worker.

A contrast of our benchmark framework against a version of our framework with talent-specific investments is below. In our benchmark case individual earnings are given by $W_{U,t}z$ when unskilled, and $W_{S,t}h_tz$ when skilled, where $W_{i,t}$ are the rental prices for skill $i = U, S$ at t. Augmented skills, h, are produced via $h_{t+1} = Bx_t^{\phi}$. With talent-specific investments, we posit that individual earnings are given by $W_{U,t}\ z$ when unskilled (as in our benchmark). However, when skilled, earnings are $W_{S,t}\ h_t(z)$, where augmented skills depend on talent: $h_{t+1}(z) = Bzx_t^{\phi}(z)$. This specification allows for the possibility that the representative household will invest more in individuals with higher talent.

The constraints in this case are:

$$\begin{aligned} C_t + I_t + N_t \int_{\hat{z}_t}^{\bar{z}} x_t(z)g(z)dz &= W_{U,t}U_t + W_{S,t}S_t + R_tK_t, && (14)\\ K_{t+1} &= K_t(1-\delta) + \frac{I_t}{p_t}, && (15)\\ U_t &= U_{t-1} + N_t \int_0^{\hat{z}_t} zg(z)dz, && (16)\\ S_t &= S_{t-1} + N_{t-1} \int_{\hat{z}_{t-1}}^{\bar{z}} h_t(z)g(z)dz, && (17)\\ h_{t+1}(z) &= Bzx_t(z)^{\phi},\ \text{all } z \in [\hat{z}_t, \bar{z}]. && (18) \end{aligned}$$

Note that the term $N_t \int_{\hat{z}_t}^{\bar{z}} x_t(z)g(z)dz$ in the left-hand side of (14) accounts for the resources devoted to skill-augmenting investments. The corresponding first-order conditions are:

$$\begin{aligned} C_t: &\quad \frac{L_t}{C_t} - \lambda_t = 0, && (19)\\ K_{t+1}: &\quad -\lambda_t p_t + \lambda_{t+1}\beta[R_{t+1} + p_{t+1}(1-\delta)] = 0, && (20)\\ x_t(z): &\quad -\lambda_t + \lambda_{t+1}\beta\ W_{S,t+1}B\phi z x_t(z)^{\phi-1} = 0,\ \text{all } z \in [\hat{z}_t, \bar{z}] && (21)\\ \hat{z}_t: &\quad \lambda_t W_{U,t}\hat{z}_t g(\hat{z}_t) + \lambda_t g(\hat{z}_t)x_t(\hat{z}_t) - \lambda_{t+1}\beta\ W_{S,t+1}Bx_t(\hat{z}_t)^{\phi}\hat{z}_t g(\hat{z}_t) = 0. && (22) \end{aligned}$$

Only the last two conditions change relative to the benchmark case. Note in particular that the first-order condition for the amount of goods invested in the skilled worker is somewhat special: We have one of such condition for *each* level of talent above the threshold $\hat{z}$.

In steady state, the last two conditions become:

$$\begin{aligned} 1 &= \beta W_S B\phi\ z\ x(z)^{\phi-1}\ \text{all } z \in [\hat{z}, \bar{z}] && (23)\\ W_U\hat{z} + x(\hat{z}) &= \beta\ W_S B\ x(\hat{z})^{\phi}\ \hat{z} && (24) \end{aligned}$$

Note that (24) is different from the benchmark case as investments for the *marginal* individual enter into the determination of the cutoff $\hat{z}$. Note also that (23) implies that investments for any individual in the skilled labor pool are given by

$$x(z) = (\beta\phi W_S)^{\frac{1}{1-\phi}} z^{\frac{1}{1-\phi}},\ \text{all } z \in [\hat{z}, \bar{z}] \qquad (25)$$

Table 3: Parameter Values for the benchmark model and the talent-specific investments model

Parameter	Benchmark	Talent-specific Investments
Discount factor (β)	0.966	0.966
Population growth rate (g_L)	0.009	0.009
Substitution elasticity ($1/(1-\rho)$)	1.50	1.50
Capital share (α)	0.33	0.33
Depreciation rate (δ)	0.074	0.074
Share of unskilled labor (μ)	0.32	0.29
Skill curvature parameter (ϕ)	0.34	0.38
Talent curvature parameter (ϵ)	0.26	0.15
Gamma distribution (θ)	17.6	17.6
Gamma distribution (κ)	27.7	27.7

Note: Entries show the values of the calibrated parameters for our benchmark economy (second column) and for the economy in which investment in skills depends on the talent level z (third column). Values for the discount factor and the depreciation rate are at the annual frequency. (Recall that the model period is 4 years.)

Table 4: Predictions of the two models for Trinidad and Tobago

	Ratio: Trinidad and Tobago to US (100)	
Variable	Benchmark Model	Talent-Specific Investments
Labor quality	85.0	85.4
Output	59.1	59.1
TFP	75.9	75.6

Hence, given prices, investments are larger for individuals endowed with higher levels of z.

In order to analyze the quantitative effects of talent-specific skill investments, we force the economy to be consistent with data on skill premium in the U.S. We accomplish this via the additional parameter $\epsilon > 0$ as in our benchmark model. Table 3 shows the values for the parameters in our benchmark model (see Table 3 in the paper) as well as in the model where investments are talent-specific. In both cases we successfully match the *same* targets in the data (the targets are summarized in Table 4 in the paper).

A simple way to assess the impact of talent-specific investments is to compute stationary equilibria for particular countries, and compare the predictions of the talent-specific investments model to those of the benchmark model. We compute the implications for two countries: Trinidad and Tobago, a middle-income country, and the Kyrgyz Republic, the poorest country in our sample. The Kyrgyz Republic has an output per worker of about 5.3% of the United States (the U.S. is richer by a factor of nearly 17). The second and third columns of Table 4 and Table 5 show the predictions of the benchmark model and of the talent-specific investments model. For both models, these experiments correspond to the *Variation in TFP and PISA Scores* case, as described in the paper.

The central finding that emerges from the table is that implied TFP differences are *similar* when skill augmenting investments are talent-dependent than when they are not.

We also compute the TFP-elasticity of output per worker in the talent-specific investments model. We find that the elasticity is 2.2 (versus 2.1 in the benchmark model). In sum, extending the model to allow for talent-specific skill investments does *not* change the main conclusions of our paper.

We note that our results are not surprising given the required changes in parameter values induced by talent-specific investments (see Table 1). In order to reproduce skill premia, share of unskilled labor and expenditures per student in tertiary education, the talent-specific investments model requires a *lower* value of

Table 5: Predictions of the two models for the Kyrgyz Republic

	Ratio: Kyrgyz Republic to US (100)	
<u>Variable</u>	Benchmark Model	Talent-Specific Investments
Labor quality	46.3	45.4
Output	5.3	5.3
TFP	30.5	30.9

ϵ than in the benchmark case, a slightly smaller value of the share of unskilled labor in production (μ), as well as a slightly larger skill curvature parameter ϕ. Intuitively, a lower value of ϵ discourages skill-augmenting investments and leads to a smaller change in output per worker in response to a TFP change. This is offset by the larger value of ϕ. Overall, these effects approximately balance out and the quantitative conclusions of the paper do not change.

4 PISA Test Scores and Earnings

We now connect our findings to empirical work on the relationship between PISA test scores and individual earnings. An important caveat is in order. Available empirical work is for only one country and when workers are still at the early stages of their working life cycle. More research on the topic is needed to establish robust conclusions.

To the best of our knowledge, the only study that relates PISA scores with earnings comes from Canada. This study uses data on earnings of 2,988 individuals in 2006 that took the PISA test in 2000.[2] Since individual earnings could be affected by background characteristics of students the authors of the Canadian study proceed by regressing log-hourly earnings of males and females separately on the reading PISA score and a vector of variables that control for family factors, school factors, employment factors, demographic factors, early work exposure and educational attaintment (secondary school graduate, college dropout, university drop out, college graduate, university graduate). Table 6 shows the coefficients associated with the PISA score for two different regressions: one in which log earnings per hour is regressed on PISA scores and another in which regressors include the vector of control variables. The coefficients describe the change in earnings associated with one standard deviation increase in PISA score.

For males, one standard deviation increase in PISA increases earnings by 0.8% (with controls) and by 2.1% (no controls). For women, the effects are stronger; a one standard deviation increase in PISA increases earnings by 4.1% (with controls) and by 5% (no controls).

In order to compare our results with the estimates in the Canadian study we proceed as follows. Using the PISA distribution for Canada we simulate 30,000 workers and then use the steady-state values of endogenous variables to compute earnings for these workers. We then regress log-earnings on the PISA scores of these simulated workers, separately for the skilled and unskilled. Our findings imply that one standard deviation in PISA scores increases earnings by about 4% for the skilled group and about 5.7% for the unskilled group. We also estimated the regression coefficients for the entire population by adding a dummy variable for skilled workers. The increase in earnings in this case is 5.2%. These findings are reassuring, as they show that our estimates are of a similar magnitude as those obtained from the Canadian study.

[2] Organisation for Economic Cooperation and Development (2010), *Pathways to Success – How Knowledge and Skills at age 15 Shape Future Lives in Canada*, Paris, OECD Books.

Table 6: Canada Study: Effect of PISA scores on Log hourly earnings

	Coefficient Associated with PISA score	
	Males	Females
No Controls	0.021	0.05
Controls	0.008	0.041