# On-Line Appendix

for

**Managers and Productivity Differences**

by

Nezih Guner, Andrii Parkhomenko and Gustavo Ventura

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## 1 Balanced Growth

Along a balanced growth path (i) growth rates are constant; (ii) the growth rate in output equals the growth rate in labor and managerial income; (iii) growth in aggregate skill investment is the same as the growth rate in output; (iv) the capital-output ratio is constant; (v) the fractions of managers and workers are constant (i.e. $z^*(t) = z^*$ for all $t$); (vi) factor prices are constant.

We find the growth rate in output per person ($g$) and initial managerial skills ($g_z$) consistent with (i)-(vi), given a growth rate in exogenous productivity ($g_A$). Specifically, we show that there is a balanced growth path if and only if initial managerial skills grow at a specific rate determined by exogenous productivity growth.

From the properties of the plant's technology, it follows that

$$1 + g = (1 + g_A)\,(1 + g_z)^{1-\gamma}(1 + g_k)^{\alpha\gamma},$$

where $g_k$ stands for the growth rate of capital per person. It follows that

$$1 + g = (1 + g_A)^{\frac{1}{1-\alpha\gamma}}(1 + g_z)^{\frac{1-\gamma}{1-\alpha\gamma}} \tag{1}$$

We proceed now to find the rate of growth of managerial skills that is consistent with a balanced-growth path. We denote by $g_z^*$ such growth rate. Note that if such path exists, then the age profile is shifted by a time-invariant factor $(1 + g_z^*)$. That is,

$$\frac{z_j(t+1)}{z_j(t)} = (1+g_z^*)$$

for all $j = 1, ...J_R - 1$. It follows that we can infer the value of $g_z^*$ from the first-order conditions for skill investments of two cohorts of age $j \leq J_R - 2$, at two consecutive dates. In particular, the first-order condition for decisions at the penultimate period of the working life cycle must hold along a balanced-growth path. From (**??**), it follows:

$$\left(\frac{1}{1+g_z^*}\right)^{\theta_1}\left(\frac{1}{1+g}\right)^{\theta_2-1} = (1+g_A^*)^{\frac{1}{1-\gamma}}\left(\frac{1}{1+g}\right)^{\frac{\gamma(1-\alpha)}{1-\gamma}}. \tag{2}$$

In deriving the expression above, we used the fact that along a balanced growth path, the rate of return is constant and that the growth in output per capita, $g$, equals the growth rate in skill investments and the growth rate in wage rates. Solving for $g_z^*$ in (2), we obtain:

$$1+g_z^* = (1+g)^{\frac{\gamma(1-\alpha)+(1-\theta_2)(1-\gamma)}{\theta_1(1-\gamma)}}\left(\frac{1}{1+g_A}\right)^{\frac{1}{\theta_1(1-\gamma)}} \tag{3}$$

Substituting (3) in (1), after algebra we obtain

$$1+g = (1+g_A)^{\psi},$$

where $\psi$

$$\psi \equiv \frac{1-\theta_1}{\gamma(1-\alpha)+(1-\theta_2)(1-\gamma)-\theta_1(1-\alpha\gamma)}. \tag{4}$$

**Comments** Several points are worth noting from the expression above. First, there is balanced growth path with positive growth in per capita output as long as $\theta_1 \in [0,1)$. Second, all the same, the growth rate in output per capita increases with $\theta_2$: as the importance of investments in the production of new skills increases, the growth rate in output per capita increases as well. Indeed, as $\theta_2 \to 0$,

$$\psi \to \frac{1}{1-\alpha\gamma}.$$

That is, the growth rate approaches the growth rate with exogenous skill investments given by the reciprocal of one minus the capital share.

Finally, as the span-of-control parameter approaches 1,

$$\psi \to \frac{1}{1-\alpha},$$

which results in the growth rate of a standard economy with constant returns in capital and labor.

# 2 Occupational Transitions

In the benchmark economy presented in detail in Section 3, each individual chooses his/her occupation, whether to be a worker or a manager, at the start of his/her life and this decision is *irreversible*. In this Appendix, we first document facts on transitions between managerial and non-managerial occupations for the U.S., and then build and calibrate a model economy that allows agents to switch between occupations. Finally, we study, as we did in sections **??** and **??**, the effects of changes in economy-wide productivity ($\bar{A}$) and the size dependence of distortions.

## 2.1 Data on Occupational Transitions

In order to compute transitions between managerial and non-managerial occupations in the United States, we use data from the Outgoing Rotation Groups of the Current Population Survey (CPS) for 1990-2010 period. Every household (address) that enters the CPS is interviewed for 4 consecutive months, then ignored (rotated out) for 8 months, and then interviewed again (rotated in) for 4 more months. As a result, it is possible to have observation on a subset of CPS sample that is one year apart. We follow a standard matching procedure, specified in Shimer (2012), based on matching households with the same identification code, as long as household members' characteristics (age, sex, race and education) are consistent between two points in time. The sample consists of individuals aged 25-64 who work at least 30 hours a week.

Based on matched households, we compute the fraction of individuals between ages 25-29, 30-34,..., 60-64 who transit from a managerial (non-managerial occupations) to a non-managerial (managerial) occupation within a year. A transition from managerial (non-managerial) to non-managerial (managerial) occupation occurs if in month $t$ a worker reports an occupation that belongs to the set of managerial (non-managerial) occupations, while in month $t + 12$ he/she reports an occupation that belongs to the set of non-managerial (managerial) occupations. The classification that we use to distinguish between managerial and non-managerial occupations is detailed in Section 2. If a worker is not observed or does not report any occupation in the year, he is excluded from the sample we use to calculate the transitions. We report average yearly transitions for 1990-2010 period.

Figure 1 shows the transitions between occupations in our data. As the figure shows, there are significant transitions between occupations from one year to the next. Each year about 4-5% of individuals with a non-managerial occupation move to an managerial occupation, while a much larger fraction, 40-50%, of individuals with a managerial occupation moves to a non-managerial occupation.

Transitions between managerial and non-managerial work can naturally change the fraction of individuals engaged in managerial work at different ages. To assess these potential changes, we compute the fraction of managers using the U.S. Census and ACS; the same data sets that we used to calculate managerial and non-managerial income profiles in Section 2. We calculate the fraction of managers averaged across four years (1990, 2000, 2005, and 2010). The fraction of managers grows with age in the first part of the working life cycle, and then becomes approximately constant. The fraction of individuals with a managerial occupation between ages 25-29 and 45-49 increases from about 7% to 11.8%. After that, the fraction of managers is relatively constant until the retirement age.

## 2.2 Model

Consider now the following version of the model economy described in Section 3. Each individual is born with a managerial ability $z$, and individuals have access to a production technology to increase their managerial ability. This technology maps the current managerial ability and investment in human capital into a managerial ability level next period.

We introduce two changes into the basic model. First, we assume that accumulation of managerial skills is *risky* as in Huggett, Ventura and Yaron (2011). At the end of each period, all individuals receive a random shock, denoted by $\varepsilon$, that determines their level of skills next period in conjunction undepreciated skills and the production of new skills. In particular for a $j$-years old individual with a current skill level $z$ and investment $x$, the next period's skill level is given by

$$z' = \varepsilon \left[(1-\delta_z)z + B(j)z^{\theta_1}x^{\theta_2}\right].$$

Second, we allow both managers and workers to accumulate managerial human capital. In particular, we assume that at the start each period, all individuals, managers ($M$) and workers ($W$), decide whether to be a manager or a worker for that period. They make this decision *before* they observe $\varepsilon$. We assume that $\varepsilon$ is an *iid,* across time and individuals, shock distributed according to a cumulative distribution function $G_o(\varepsilon)$, $o \in \{W, M\}$. Once the individuals make their occupation choice, they decide how much to consume, how much to save and how much to invest to enhance their skills, $x$. They make all these decisions again before they observe $\varepsilon$. After the investment decisions are made, $\varepsilon$ is realized and the individuals enter next period with their updated level of human capital. Then they again make an occupational choice decision and so on.

In this environment, although managerial skills do not affect the current income of workers, as they simply earn $w$, they still have an incentive to invest in their skills as a favorable $\varepsilon$ shock in the future can make them switch occupations next period. A manager, on the other hand, can decide to become a worker if his/her $\varepsilon$ was too low last period. We assume that switching occupation has no monetary or utility cost.

Consider the problem of an age-$j$ individual. At the start of the of the period, given his current skills ($z$) and assets ($a$), this individual decides whether to be a manager to a worker:

$$V(a, z, j) = \max\left\{V^M(a, z, j), V^W(a, z, j)\right\}.$$

The value of being a manager $V^M(a, z, j)$ is given by

$$V^M(a, z, j) = \max_{c, a', x}\left\{u(c) + \beta \int V(a', z'(\varepsilon), j+1) dG_M(\varepsilon)\right\},$$

subject to

$$c + a' + x \leq \pi(z, r, w) + (1 + r)a,$$

and

$$z' = \varepsilon\left[(1 - \delta_z)z + B(j) z^{\theta_1} x^{\theta_2}\right],$$

where $\pi(z, r, w)$ is the profits of managers as defined by equation 6 in Section 3.1.

The value of being a worker $V^W(a, z, j)$, on the other hand, is given by

$$V^W(a, z, j) = \max_{c, a', x}\left\{u(c) + \beta \int V(a', z'(\varepsilon), j+1) dG_W(\varepsilon)\right\},$$

subject to

$$c + a' + x \leq w + (1 + r)a,$$

and

$$z' = \varepsilon\left[(1 - \delta_z)z + B(j) z^{\theta_1} x^{\theta_2}\right]$$

## 2.3 Parameter Values

We follow the same calibration strategy as described in Section 4. In addition to the parameters listed in Table 1, we need to specify the functional forms for $G_M(\varepsilon)$ and $G_W(\varepsilon)$. We assume that both distributions are log-normal with mean zero and variances denoted by $\xi_M$ and $\xi_W$. In the model economy, these variances have implications for the fraction of managers in the labor force at each age as well as the relative age-earnings profile of managers. As a result, in order to calibrate these parameters we select two new targets: i) the fraction of managers at age 60-64 relative to the fraction of managers at age 25-29 and ii) an additional moment from the age-earnings profile – the relative earnings at age 50-54 (recall that relative incomes at ages 40-44 and 60-64 were already among the targets in Table 2). Table A3 presents the calibrated parameters of the model with occupational transitions. Table A4 compares the data and the model moments. The model captures endogenously the growth in the number of managers by age. Both the model and the data, the fraction of population with fraction of individuals with a managerial occupation increases by a factor of 1.63 between ages 25-29 to 60-64. Figure 2 shows the relative age-earnings profiles of managers in the model and the data. The model matches very well the age-earnings profiles of managers.

With a few exceptions parameter values in Tables 1 and A3 are quite similar. In particular, the span of control parameter $\gamma$ is larger in the economy with occupational transitions. The volatility of skill shocks is larger for workers than it is for managers: the standard deviation for workers is $\xi_W = 0.335$ while the standard deviation for managers is $\xi_M = 0.215$. Since individuals are risk-averse and there is no explicit age-dependent preference for occupation, a smaller variance of shocks to managers' skills is needed to be consistent with the fact that the fraction of managers in the workforce grows by 63% from ages 25-29 to ages 60-64.

## 2.4 Results

To what extent do our baseline results change when we allow occupational changes over the life cycle? We now revisit the analysis of Section 5.4 and check how the economy reacts to changes in exogenous productivity and size dependency of the distortions. We report our findings in Tables A5 and A6.

We first proceed to gradually lower the exogenous TFP ($\bar{A}$) from the benchmark value of 1 to 0.7. The effects of lower $\overline{A}$ values on aggregate output is very similar to ones we obtain for an economy without managerial transitions – compare Table 3 and Table A5. Relative earnings growth declines with a reduction of economy-wide productivity across steady states, although by a smaller magnitude than under the benchmark model.

These findings show the interaction of opposing effects. On the one hand, in the model economy with occupational transitions, individuals have an additional incentive to invest in skills given by skill-accumulation risk and the occupational choice it facilitates. As a result, skill investment does not decline as rapidly in response to reduction in $\bar{A}$ as in the baseline model – compare Table 3 and Table A5. Therefore, the response of managerial quality and relative earnings growth to exogenous productivity is more muted than in the baseline analysis. On the other hand, the fraction of managers in the labor force is almost constant for all levels of $\bar{A}$, whereas it rises slightly in the baseline model. The combination of these effects results in the response of output to $\bar{A}$ which is almost identical to the one in the baseline model.

We then gradually increase the size dependency of the distortion ($\tau$) from the benchmark value of 0 to 0.08. The effects on output, mean establishment size, relative earnings growth, fraction of managers, and managerial quality are very similar to those found for the baseline model – compare Table 4 and Table A6. As in the experiment with $\bar{A}$, the response of skill investment is much smaller compared to the baseline model. Clearly, size-dependent distortions reduce managers' incentives to invest in skills in order to earn higher managerial rents. However, individuals still use skill investment as an insurance against negative skill shocks. On top of that, given the option value of an occupational switch, workers aspiring to become managers keep investing in skills even at high levels of $\tau$.

Table A3: Parameter Values (annualized)

| Parameter | values |
|---|---|
| Population Growth Rate ($n$) | 0.011 |
| Productivity Growth Rate ($g$) | 0.025 |
| Depreciation Rate ($\delta$) | 0.040 |
| Importance of Capital ($\alpha$) | 0.386 |
| Returns to Scale ($\gamma$) | 0.844 |
| Mean Log-managerial Ability ($\mu_z$) | 0 |
| Dispersion in Log-managerial Ability ($\sigma_z$) | 3.01 |
| Discount Factor ($\beta$) | 0.931 |
| Skill accumulation technology ($\theta$) | 0.862 |
| Skill accumulation technology ($\delta_\theta$) | 0.067 |
| Skill accumulation technology ($\theta_1$) | 0.686 |
| Skill accumulation technology ($\theta_2$) | 0.461 |
| Skill accumulation technology ($\delta_z$) | 0.008 |
| Std deviation of skill shocks, managers ($\xi_M$) | 0.215 |
| Std deviation of skill shocks, workers ($\xi_W$) | 0.335 |

Note: Entries show model parameters calibrated for the model with occupational transitions. See text for details.

Table A4: Empirical Targets: Model and Data

| Statistic | Data | Model |
|---|---|---|
| Mean Size | 17.9 | 17.7 |
| Capital Output Ratio | 2.33 | 2.33 |
| Relative Earnings Growth ($\hat{g}$) (40-44/25-29) | 0.17 | 0.16 |
| Relative Earnings Growth ($\hat{g}$) (50-54/25-29) | 0.22 | 0.23 |
| Relative Earnings Growth ($\hat{g}$) (60-64/25-29) | 0.22 | 0.22 |
| Fraction of Managers (60-64/25-29) | 1.63 | 1.63 |
| *Fraction of Establishments* | | |
| 1-9 workers | 0.725 | 0.757 |
| 10-20 workers | 0.126 | 0.108 |
| 20-50 workers | 0.091 | 0.076 |
| 50-100 workers | 0.032 | 0.028 |
| 100+ workers | 0.026 | 0.031 |
| *Employment Share* | | |
| 1-9 workers | 0.151 | 0.163 |
| 10-20 workers | 0.094 | 0.092 |
| 20-50 workers | 0.164 | 0.142 |
| 50-100 workers | 0.128 | 0.120 |
| 100+ workers | 0.462 | 0.483 |

Note: Entries show the empirical targets used in the quantitative analysis and the model's performance in the model with occuppational transitions. The fraction of establishments with 1-9 and 100+ workers, and the employment shares with 1-9 and 100+ workers are explicit targets. See text for details.

Table A5: Effects of Economy-Wide Productivity

| Economy-Wide Productivity | $\overline{A}=1$ | $\overline{A}=0.9$ | $\overline{A}=0.8$ | $\overline{A}=0.7$ |
|---|---|---|---|---|
| Statistic | | | | |
| Output | 100 | 84.5 | 68.6 | 55.6 |
| Mean Size | 17.7 | 17.7 | 17.6 | 17.3 |
| Investment in Skills | 100 | 93.5 | 85.9 | 80.8 |
| Investment in Skills (% Output) | 8.1 | 8.9 | 10.1 | 11.7 |
| Number of Managers | 100 | 99.7 | 100.5 | 102.0 |
| Managerial Quality | 100 | 98.0 | 94.6 | 91.2 |
| Employment Share (100+) | 0.48 | 0.48 | 0.47 | 0.46 |
| Relative Earnings Growth ($\hat{g}$) | 0.23 | 0.19 | 0.17 | 0.10 |

Note: Entries show the effects on displayed variables associated to exogenous reductions in the level of economy-wide productivity ($\bar{A}$) across steady states. Column 2 reports benchmark values ($\bar{A}=1$). Columns 3-5 report the changes emerging from reducing $\bar{A}$ below the benchmark value. See text for details.

Table A6: Effects of Size-Dependent Distortions

| Size Dependency ($\tau$) | 0 | 0.02 | 0.04 | 0.06 | 0.08 |
|---|---|---|---|---|---|
| Tax Wedge $\left(\frac{1-T(5\overline{y})}{1-T(y)}\right)$ | 1 | 0.97 | 0.94 | 0.91 | 0.88 |
| Statistic | | | | | |
| Output | 100.0 | 93.0 | 83.7 | 78.6 | 73.3 |
| Mean Size | 17.7 | 13.0 | 10.1 | 8.1 | 6.8 |
| Investment in Skills | 100.0 | 87.1 | 78.6 | 75.1 | 72.8 |
| Investment in Skills (% Output) | 8.1 | 7.5 | 7.6 | 7.7 | 8.0 |
| Number of Managers | 100.0 | 136.1 | 174.6 | 217.5 | 261.6 |
| Managerial Quality | 100.0 | 72.2 | 54.9 | 43.7 | 36.0 |
| Employment Share (100+) | 0.48 | 0.34 | 0.22 | 0.13 | 0.07 |
| Relative Earnings Growth ($\hat{g}$) | 0.23 | 0.10 | 0.02 | -0.01 | -0.05 |

Note: Entries show the effects on displayed variables associated to size-dependent distortions across steady states. Column 2 reports benchmark values ($\tau=0$). Columns 3-6 report the changes emerging from increasing the size dependency of distortions. See text for details.

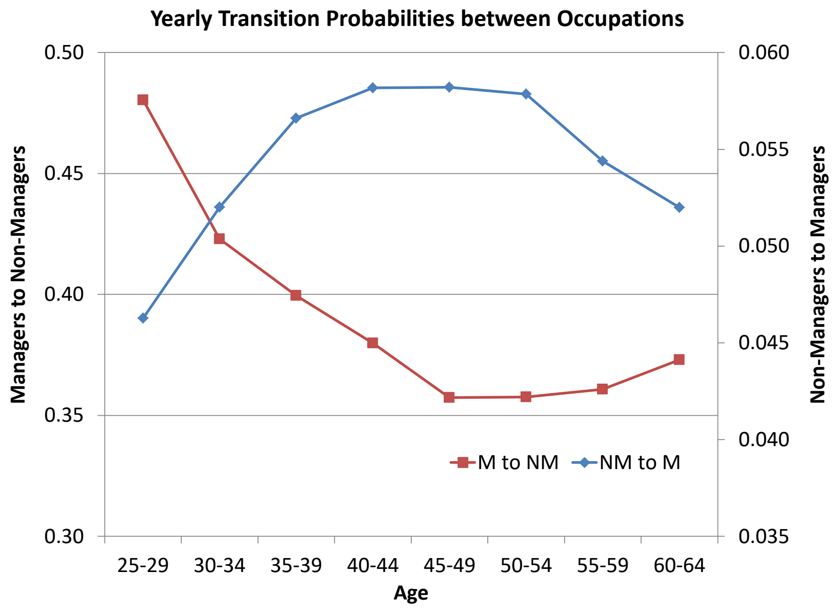


Figure 1


Figure 2